FREQUENCY ANALYSIS OF MAXIMUM FLOWS RECORDED IN THE UPPER JURUÁ RIVER BASIN, ACRE, BRAZIL

ANÁLISE DE FREQUÊNCIA DE VAZÕES MÁXIMAS REGISTRADAS NA BACIA HIDROGRÁFICA DO ALTO RIO JURUÁ, ACRE, BRASIL

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ABSTRACT

The risk associated with extreme floods is manageable and can be reduced, as it relates both to the probability of occurrence of a dangerous event and to the expectation of losses caused by it. In this sense, frequency analysis through probabilistic modeling of hydrological variables is an important tool. The present study aimed at investigating the probabilistic model that best fits the series of maximum flows recorded in the Upper Juruá River basin (Acre, Brazil), highlighting the probability associated with the recurrence of the main quantifying reference. It was concluded that generalized extreme value (GEV) and two-parameter lognormal (LN2) are the recommended distributions for Cruzeiro do Sul and GEV for Marechal Thaumaturgo, whose decision was supported by the Kolmogorov-Smirnov and Anderson-Darling tests. LN2 proved to be more rigorous with a return time of more than 50 years for flows in Cruzeiro do Sul.

Keywords: Extreme floods; probabilistic models; frequency analysis.

RESUMO

O risco associado a cheias extremas é passível de gerenciamento e pode ser reduzido, pois se relaciona tanto com a probabilidade de ocorrência de um evento perigoso quanto com a expectativa de perdas causadas por ele. Neste sentido, a análise de frequência através da modelagem probabilística de variáveis hidrológicas é uma ferramenta importante. O presente estudo se dedicou a investigar o modelo probabilístico que melhor se ajusta em séries de vazões máximas registradas na bacia hidrográfica do Alto Rio Juruá (Acre, Brasil), destacando a probabilidade associada à recorrência dos principais quantis de referência. Concluiu-se que a generalizada de valores extremos (GEV) e a log-normal de dois parâmetros (LN2) são as distribuições recomendadas para Cruzeiro do Sul e a GEV para Marechal Thaumaturgo, cuja decisão foi amparada pelos testes de Kolmogorov-Smirnov e Anderson-Darling. A LN2 se mostrou mais rigorosa com tempo de retorno superior a 50 anos para vazões em Cruzeiro do Sul. **Palavras-chave:** cheias extremas, modelos probabilísticos, análise de frequência.

1. INTRODUCTION

Natural disasters often affect socioeconomic systems, imposing catastrophic impacts over different forms of damage. Hydrological disasters stand out for the way they reach the most vulnerable areas and are inserted in a category particularly triggered, to a large extent, by processes that imply extreme floods [1].

In the scenario of changes in climatic patterns, both at regional and global scale, many studies point to an increase in the record of natural events capable of promoting changes in hydrological patterns and, consequently, increasing the risk associated with extreme flood events. In this direction, most researchers agree to the conclusion that, among so many factors, actions caused by anthropic activities are among the main causes [2], [3], [4], [5].

Floods are commonly caused by extreme flood events and have been affecting humanity since the so-called Classical World. Historically, the interplay between cities and extreme floods has not been balanced, especially in contemporary society, whose process of disordered urbanization, along with other factors potentiate the risk associated with increasingly harmful events [1], [6], [7].

The adverse impacts of extreme floods are usually severe, especially in the Brazilian regional context, whose spatial distribution of records presents significant variability [8]. Under crisis conditions, solutions aiming at mitigating impacts are valuable and have become central elements in search for improvement by humanity, in order to balance the relations of the natural and social environments [9].

Preventing extreme natural events from occurring are beyond human capacity. However, the risk associated with maximum flood events is manageable and can be reduced, taking into account that the risk is related both to the probability of occurrence of a dangerous event, as well as to the expectation of losses caused by it [1], [10], [11].

In this sense, frequency analysis through probabilistic modeling of hydrological variables, such as maximum floods, is an indispensable tool regarding extreme events associated with the probability of their occurrences, being of help to the practices of planning and management of water resources [12]. According to [13], the choice of the probabilistic model is the base-stage for frequency analysis, being a recurrent subject in the area literature.

Based on these considerations, the aim of the present study is to investigate the probabilistic model that best fits the series of maximum flows recorded in the Upper Juruá River basin, State of Acre, Brazil. Furthermore, it is another objective of the research, to develop the frequency analysis for the variable under study, highlighting the probability associated with the recurrence of the main reference quantiles.

2. MATERIAL AND METHODS

2.1 STUDY AREA

The study focused on the delimited area of the Upper Juruá River sub-basin, concentrated at the limits of the state of Acre, Brazil (Figure 1). The region is marked by the predominance of large rivers and a territory with a vast expanse of forest. According to Köppen,

the climate of the region is equatorial, warm and humid (Af), showing an average annual temperature around 24.5 oC. The region presents a pattern with a dry period (May to September) and a remarkably rainy period (October to April), with annual rainfall averages of approximately 2,000 mm [14], [15].

The flow regime recorded in the study area is similar to that presented throughout the state of Acre, with a period of low flow values and another when the highest annual flows are recorded, corresponding to the dry and rainy periods, respectively. In the popular culture, the dry period and low flow values is known as the "Amazonian summer", whereas the rainy season and higher flows in the watercourses is known as the "Amazonian winter". In the period of high rainfall, it is common the occurrence of overflow the main channel of rivers and consequent invasion of their floodplains [15].



Figure 1: Location of the basin of the Upper Juruá River, Acre, and gauging stations analyzed.

2.2 HYDROLOGICAL SERIES

In the present study, we used the data of maximum annual daily flows (MADF), recorded in the gauging stations of the National Hydrometeorological Network (NHN), under codes 12370000 and 1250000, located on the Juruá River, respectively in the municipalities of

Marechal Thaumaturgo (8°56'S; 72°47'W) and Cruzeiro do Sul (7°38'S; 72°39'W) (datum WGS84), both operated by the Brazilian National Water Agency (ANA) (Figure 1).

For the station located in Marechal Thaumaturgo, were considered the data for the period from January 1982 to December 2011, whereas for Cruzeiro do Sul were used the data from January 1968 to December 2011. Only the values already consisted of the variable under study were considered, which are included in the ANA database through the HidroWeb.

It is important to note that there are two other similar stations in the region, which are not being considered in the present study: one located at the Foz do Breu community (border with Peru), and the other located at Porto Walter. The exclusions were motivated by significant failures presented in the data series of the Foz do Breu station, which could compromise the results of the analysis, whereas the data series of the other station covers a very short period of records and could equally compromise the adjustment of probabilistic models.

2.3 STATISTICAL ANALYSIS

The frequency analysis of hydrological data presents a fundamental step, the choice of the probabilistic model, which, in turn, should lead to estimates with accuracy and precision. This choice requires great care on decision. [13] point out that the search for the distribution that best fits the frequency analysis data has been a recurring issue, especially regarding the determination of the probability distribution for maximum flow records.

There are several distributions of probabilities that could be used for this purpose, among the most relevant are the Generalized of Extreme Values (GEV), Gumbel for Maxima (GUM), Log-normal of 2 parameters (LN2), which will be used in the present study [12], [16].

These distributions are among those recommended by [13], who include them among the recommended for the adjustment of maximum flow rates. Yet, [17] suggests the use of the distribution of Gumbel, in particular, as one of the most used probability distributions in hydrological studies, with numerous applications, especially in the study of maximum flow rates, in which it presented satisfactory results. As described by [16], the probability density function (pdf) for the GEV distribution is expressed by Equation 1:

$$f_X(x) = \frac{1}{\sigma} \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{-\left(\frac{1 + \xi}{\xi} \right)} \cdot exp \left\{ - \left[1 + \xi \left(\frac{1 - \mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\}$$
(1)

where μ , σ , and ξ are the parameters of position, scale, and shape, respectively. The Gumbel pdf (GUM), which contains the position and scale parameters, is expressed by Equation 2:

$$f_X(x) = \frac{1}{\sigma} exp\left\{-\frac{x-\mu}{\sigma} - exp\left(-\frac{x-\mu}{\sigma}\right)\right\}$$
(2)

In turn, the Log-normal distribution (LN2), with the position and scale parameters, has the pdf represented by Equation 3:

$$f_X(x) = \frac{1}{x \,\sigma_{\ln(X)} \sqrt{2\pi}} exp\left\{-\frac{1}{2} \left(\frac{\ln(X) - \mu_{\ln(X)}}{\sigma_{\ln(X)}}\right)\right\}$$
(3)

In practice, the cumulative probability function (CPF) is commonly used, which, according to [18], is obtained by the Equation 4:

$$F_X(x) = \int_{-\infty}^x f_X(x) \, dx \tag{4}$$

Concerning the adjustment, [12] highlight the Momentum Method (MM), the L-Momentum Method (ML), and Maximum Likelihood (ML), which influence the degree of adjustment of the pdf to the observed data. For the present study, the ML was used, which is used in the adjustment of probabilistic models of hydrological variables. Only the GEV model referring to Marechal Thaumaturgo station was adjusted using the MM, meeting basic assumptions [16], [19], [20].

After determining the probabilistic model, it is possible to verify the quality of the adjustment of the theoretical distribution to the series of observations, whose decision is based on the adherence tests, with the Kolmogorov-Smirnov test (KS), and Anderson-Darling test (AD) among the most used in the literature and adopted in the present study. These tests served as the criterion to choose the probability distribution model used to determine the amounts of flood quantiles, associated with different probability levels [13], [12], [16].

The routine for determining the statistics of each adherence test, as well as the hypotheses and decision about them are well described by [21], whereas [12] reports on the characteristics of each test, as well as on the conclusions obtained in studies that used such tools. [12] states that, despite several conclusions regarding greater or lesser accuracy of each of these tests, there is no consensus on the best criterion for choosing the probability model that provides better fit and less uncertainty in the decision.

Based on the decision on the best performing probability distribution model, it should be estimated the maximum flood quantiles associated with the main values of return time and hydrological risk associated with the probability of equality or the overcoming of a given event. These values are obtained from the inversion of CPF of the probability functions described in equations (1), (2), and (3), applying the value of the return time, T, which is determined from $F_X(x_T)$ and the value associated to a reference quantile x_T , and expressed by Equation 5:

$$T(x_T) = \frac{1}{P(X > x_T)} = \frac{1}{1 - F_X(x_T)}$$
(5)

In turn, the hydrological risk, R, defined as the probability that a reference event would be matched or overcome at least once over N years, is defined by Equation 6:

$$R = 1 - \left[1 - \frac{1}{T(x_T)}\right]^N \tag{6}$$

It should be noted that the main assumption for the classical frequency analysis of extreme values, as it is proposed in the present study, is the non-rejection of the hypothesis of stationarity in the series of observations. This assumption was met according to the conclusions presented by [15]. In addition to this baseline assumption, these authors concluded that the hypotheses of randomness, independence, and homogeneity were also not rejected.

The practical application of the highlighted tests requires extensive routine and calculations until decision-making on the hypotheses tested, therefore, computational tools were used, such as the Local Frequency Analysis of Annual Events (LFAAE) software and the functional softwares of the platform "R Project for Statistical Computing" [20], [13].

3. RESULTS AND DISCUSSION

The analyzed series present high values in their records of maximum daily annual flows. For the period under analysis, the average recorded values in Cruzeiro do Sul was $3,243.9 \text{ m}^3/\text{s}$, while for Marechal Thaumaturgo the records averaged $2,319.9 \text{ m}^3/\text{s}$.

In this sense, [15] evaluated these records under the direction of temporal trend analysis and verified that the flow regime presented an upward rate of variation in Cruzeiro do Sul and a descending profile in Marechal Thaumaturgo, although the authors concluded that this variation was not statistically significant. As a physical implication of these findings, the authors concluded that the flow regime in the Upper Juruá basin did not show significant changes over time, at least for the time window that was studied. Regarding the high flow values recorded, the authors considered as a remarkable characteristic of the rivers of the Amazon, which present a distinct surface flow regime.

Returning to the main objective of the present research, frequency analysis, the methodological elements outlined in this study allowed it to conclude in support of the hypothesis that the probabilistic behavior of the maximum annual daily flows can be modeled by the Generalized distribution of Extreme Values (GEV) and Log-normal of 2 parameters (LN2) for the records of Cruzeiro do Sul, and by GEV for the data obtained for Marechal Thaumaturgo. Table 1 shows the values of the parameters adjusted to the probabilistic models listed as candidates, as well as the statistical values of the Kolmogorov-Smirnov (KS) and the Anderson-Darling (AD) adherence tests.

 Table 1: Parameters of the adjustment of the theoretical distribution to the data of daily maximum

 flow rates annually and values of the statistics of adherence tests.

Gauging Station	Distribution	Distribution Parameters			Statistic adherence	Statistics of adherence tests	
		Position	Scale	Shape	KS ¹	AD^2	
Cruzeiro do Sul	GEV	3,067.041	500.576	0.278	0.079	0.239	
	GUM	2,993.969	497.078		0.113	1.052	
	LN2	8.072	0.159		0.081	0.198	
Marechal Thaumaturgo	GEV	2,201.000	396.900	0.371	0.149	0.748	
	GUM	2,129.038	362.097		0.146	1.043	
	LN2	7.736	0.168		0.164	0.767	
1							

 ${}^{1}D_{44;0.05} = 0.205; {}^{1}D_{30;0.05} = 0.242$

 $^{2}AD_{0.05} = 0.752$ (log-normal); $^{2}AD_{0.05} = 0.757$ (Gumbel e GEV).

Based on the KS test, the decision was not to reject the hypothesis that the data on question can have their probability distribution adjusted by the three models set and verified, both for the records referring to the fluviometric stations of Cruzeiro do Sul and Marechal Thaumaturgo (Dmax < Dcrit). On the other hand, according to the results of the Anderson-Darling test (AD), it was decided to reject the Gumbel distribution (GUM) to model the probabilistic behavior of the annual maximum daily flow data recorded in the Cruzeiro do Sul gauging station. Likewise, the GUM and LN2 models were rejected for the records of Marechal Thaumaturgo station (AD > Dcrit).

Regarding the decisions expressed regarding adherence to the models under test, [16] and [18] suggested that both AD and KS tests are based on differences in the empirical and theoretical cumulative probabilities. According to the authors, the power of the KS test has decreased accuracy in the tails of the distributions because the number of observations and estimation errors, whereas the AD test pursues to more strongly weigh the tails of the distributions, thus can drastically influence the quality of the adjustment.

The Gumbel distribution is widely used in studies of frequency analysis of hydrological variables, with numerous reports of positive results in the literature [22] and [23]. However, in this study, it did not present adherence to the data of the two stations analyzed, according to the AD test. This decision may be associated with the rigor established by this test, since from the graphical analysis of data adjustment to the model in question, there was no important difference between experimental data and modelled curve. Figure 2 shows the adjustment of the GUM distribution to the data analyzed for both stations of Cruzeiro do Sul and Marechal Thaumaturgo, in addition to the other two models tested.



Figure 2: Adherence of GEV, GUM and LN2 models to the data of maximum annual flow rates in the gauging stations of Cruzeiro do Sul and Marechal Thaumaturgo.

Concerning the recommended distributions to model the maximum flow data from Cruzeiro do Sul (GEV and LN2) and Marechal Thaumaturgo (GEV) stations, the decision is supported both in the results from the tests used in this work, as well as in results obtained in previous studies, which report similarly satisfactory outcomes. Thus, [24] concluded in favor of GEV as the compatible distribution to adjust the annual maximum precipitation data in the southern region of Minas Gerais, whereas the LN2 model was suggested in the conclusions presented by [13], indicating that this model is adequate to adjust data of Brazilian hydrological records.

Based on the parameters according to the models recommended for the hydrological records under study, the cumulative probability functions (CPF), obtained from the estimated pdf, become important elements to estimate the probability of overcoming an event capable of promoting damage. Additionally, the occurrence of maximum flood events can be related to the estimated return time. Table 2 shows the values of maximum annual daily flow rates for different return times (T), estimated according to the CPF of each of the recommended distributions. Similarly, it is trivial to estimate MADF values related to other T values.

Table 2: Estimated maximum daily flow (m³/s) for Cruzeiro do Sul and Marechal Thaumaturgo according to the generalized probability distributions of Extreme Values and Log-normal values of 2 parameters, at different return times (years).

T (years)	Cruzeire	o do Sul	Marechal Thaumaturgo	
	GEV	LN2	GEV	
5	3,681.0	3,662.2	2,657.6	
10	3,904.4	3,927.5	2,806.6	
25	4,127.6	4,231.7	2,944.3	
50	4,259.1	4,440.6	3,019.3	
100	4,366.5	4,637.3	3,076.7	
500	4,547.6	5,062.6	3,164.1	
1000	4,603.7	5,236.2	3,188.3	

The results expressed in Table 2 show that the values estimated by the GEV and LN2 distribution for the data for the Cruzeiro do Sul station are similar, especially for the return time less than 50 years. For higher values, the LN2 distribution becomes more rigorous, associating higher flow values in relation to those estimated by GEV for the same return time. From the point of view of planning in the face of the impacts of extreme events, in a project aiming at a return time of more than 50 years, the most logical decision is that the values referenced by LN2 would produce a lower estimated risk (expectations of losses are the same). Data plotted in Figure 3, in addition to confirming this conclusion, allows to estimate the maximum flow quantiles (m3/s) associated with other values of return time for both Cruzeiro do Sul and Marechal Thaumaturgo.



Figure 3: Estimated maximum flow quantile as a function of return time for Cruzeiro do Sul and Marechal Thaumaturgo.

In this sense, [16] presented similar conclusions and suggested that the results of statistical extrapolation tend to diverge according to the model adopted, especially for higher recurrence times. The author also suggests that, although for urban micro-drainage works or for agricultural projects, for example, low values are generally attributed for the return time, there are projects that would require more restrictions regarding the reference flow values, especially for long-term planning projects.

For projects that require less probability of overcoming the reference quantile (longer return time), it is recommended to investigate with more care the distributive model to estimate the reference values. Such approach is essential for decision-making regarding several strategic

areas, especially in the planning of activities aimed at mitigating the impacts associated with extreme hydrological events [25] and [16].

4. FINAL CONSIDERATIONS

The results obtained in the present study were comprehensive enough to allow an appropriate discussion on the subject proposed and, according to the defined objective, allowed us to conclude that:

- The probabilistic behavior of the maximum annual daily flows (MADF) can be modeled by using the Generalized Distribution of Extreme Values (GEV) and Log-normal of 2 parameters (LN2) for the records of the gauging station of Cruzeiro do Sul and by using the GEV for the data of Marechal Thaumaturgo;
- The Anderson-Darling adherence test (AD) was more rigorous and allowed a more logical decision about the models selected for the adjustment of the analyzed flow data;
- For return time (T) greater than 50 years the LN2 distribution becomes more rigorous, associating higher flow values in relation to those estimated by GEV for an equal T value.

Although obtaining favorable results and important conclusions, it is expected that the approach used in this research may help in decision-making in the context of managing extreme hydrological events, as well as to support new research on the subject, especially in the regional context.

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